# Why is the shape of the Laffer curve for consumption tax different from that for labor income tax? \*

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#### Abstract

It is well known that the equivalence between consumption and labor income taxes holds. However, recent macroeconomic studies on the Laffer curves (tax revenue curves) that use dynamic general equilibrium models find that the Laffer curve for consumption tax might not be hump-shaped, but monotonically increasing, whereas that for labor income tax is hump-shaped. This study investigates the cause of the difference in the shapes of the two Laffer curves by decomposing the effects of an increase in a tax rate on the tax base into two parts: (i) the effects on the relative price of leisure (RPL), and (ii) the substitution and income effects. It is shown that the first effect with respect to the consumption tax rate is completely different from that with respect to the labor income tax rate; meanwhile, the second effect is common among the taxes and depends on the functional form of the utility. The elasticity of the RPL from an increasing consumption tax rate is at most 1, whereas it can be infinity in the case of labor income tax.

**Keywords:** Laffer curve; tax revenue; consumption tax; labor income tax; relative price of leisure

JEL classifications: E62; H20; H30

### **1** Introduction

In the public finance literature, it is well known that the equivalence between consumption and labor income taxes holds. This implies that any equilibrium allocation (like output, labor supply, and consumption) in an economy with a consumption tax can also be achieved in an economy with a labor income tax. However, the equivalence on the tax revenues between consumption and labor income taxes does not hold. Recent macroeconomic studies on the Laffer curves (tax revenue curves) that use dynamic general equilibrium models find that the Laffer curve for consumption tax is not hump-shaped, but monotonically increasing, whereas the Laffer curve for labor income tax is humpshaped.

The main objective of this study is to investigate the theoretical cause of this difference in the shapes of the Laffer curves for consumption and labor income taxes. For this purpose, this study decomposes the effects of an increase in a tax rate on the tax base into two parts: (i) the effects on the relative price of leisure (RPL), and (ii) the substitution and income effects. It is shown that the first effect with respect to the consumption tax rate is completely different from that with respect to the labor income tax rate; meanwhile, the second effect is common among the taxes and depends on the functional form of the utility. The elasticity of the RPL from an increasing consumption tax rate is at most 1, whereas it can be infinity as the tax rate increases in the case of labor income tax. If a 1% increase in the tax rate reduces the tax base by more than 1%, an increase in the tax rate will reduce the tax revenue. This result then implies that the shape of the Laffer curve for consumption tax depends on the functional form of utility, whereas the Laffer curve for labor income tax is hump-shaped.

The baseline model is a simple static frictionless general equilibrium model. However, the assumption of a static economy is not crucial. The main result is applicable to a dynamic model with investment. The introduction of frictions and distortions might change the shape of Laffer curves because it generates the labor wedge, that is the wedge between the marginal rate of substitution and the marginal product of labor. However, from the viewpoint of this study, the labor wedge, that are generated by frictions and distortions, can be collectively understood as an additional source that changes the RPL.

In the baseline model, the tax revenue is used as the lump-sum transfer to the household. However, even in the case where the tax revenue is used as the government consumption, the main result is applicable. The difference in the elasticities of the RPL is the key for the difference in the shapes of the Laffer curves for consumption and labor income taxes.

The representative-agent is considered in the baseline model. The assumption of representative-agent is not crucial for the difference in the shapes of the Laffer curves for consumption and labor income taxes. Even if agents are heterogenous, each individual faces the RPL in the consumption–labor supply choice. Then, the difference in the elasticities of the RPL with respect to two taxes is still the key. One might think that the transfer scheme has important effects in a heterogenous-agent model. However, the transfer has a positive income effect, that increases consumption and decreases labor supply. Then, the Laffer curve for consumption tax is more unlikely to be hump-shaped, while that for labor income tax is more likely to be hump-shaped.

This study closely relates to the literature on the Laffer curves that uses dynamic general equilibrium models. Schmitt-Grohè and Uribe (1997), Trabandt and Uhlig (2011, 2013), Holter, Krueger, and Stepanchuk (2019), and Nutahara (2015) each found that the Laffer curve for labor income tax is hump-shaped, whereas Trabandt and Uhlig (2011, 2013), Holter, Krueger, and Stepanchuk (2019), Kobayashi (2014), Nutahara (2015), and Fève, Matheron, and Sahuc (2018) found that the Laffer curve for consumption tax is not hump-shaped. The main result points to why the two Laffer curves of these studies are different. The current study also relates to that of Hiraga and Nutahara (2019), who found that the shape of the Laffer curve for consumption tax is sensitive to the functional form of the utility.<sup>1</sup> While the concern of Hiraga and Nutahara (2019) differs from that of the current study, their findings are consistent with the results herein.

The current study also relates to the literature on tax structure and economic activity; this body of literature includes the study of Diamond and Mirrlees (1971). More recently, Knellera, Bleaney, and Gemmell (1999), Arnold (2008), and Johansson et al. (2008), empirically found that in terms of economic growth, consumption tax is less harmful than personal income tax. Nguyen, Onnis, and Rossi (2017) also found that consumption taxes are less distortive than income taxes by the proxy VARs. Hansen and Imrohoroglu (2018) found that in Japan, the replacement of income tax with consumption tax improves output and welfare; they determined this through the use of a dynamic general equilibrium model. According to the main finding of the current study, the elasticities of the RPL with respect to consumption is at most 1, whereas that with respect to labor income tax could be infinity. This could serve as a possible theoretical explanation for their results.<sup>2</sup>

The remainder of the current paper is organized as follows. Section 2 introduces the baseline model. Section 3 shows the main results. Section 4 discusses about the results. Section 5 concludes.

### 2 Model

For analytical simplicity, a simple static and frictionless general equilibrium model with representative households and competitive firms is considered.

<sup>&</sup>lt;sup>1</sup>Baydur and Yilmaz (2017) show that the Laffer curve for a value-added tax (VAT) can be humpshaped if one considers home production.

<sup>&</sup>lt;sup>2</sup>In a distorted economy, a labor income tax might be less harmful than a consumption tax. For example, Nishiyama and Smetters (2005) show that a progressive income tax is better than a flat consumption tax in a heterogenous-agent OLG model with uninsured income risk and life span shocks. Their result is beyond the scope of this paper.

The representative households supply labor *n* to firms and earn a real wage rate *w*. They also receive government transfers *s*. Let  $\tau^c$  and  $\tau^n$  denote consumption and labor income taxes, respectively, and suppose that  $\tau^c \ge 0$  and  $0 \le \tau^n < 1$ . The budget constraint of households is

$$(1 + \tau^c) c \le (1 - \tau^n) wn + s,$$
 (1)

where c denotes consumption.

The firms are perfectly competitive. Their production function is linear in labor input:

$$y = n, (2)$$

where y denotes output.

The government budget constraint is

$$s = \tau^c c + \tau^n w n. \tag{3}$$

Since there is no investment and there are no government purchases, the resource constraint of this closed economy is

$$y = c. (4)$$

Finally, the utility function of the household is U(c, n), and the standard assumptions are applied:  $U_c > 0$ ,  $U_{cc} < 0$ ,  $U_n < 0$ , and  $U_{nn} \le 0$ .

### 3 Main Results

It is useful to consider necessary conditions for hump-shaped tax revenue curves, as in Lemma 1.

**Lemma 1.** A necessary condition for a hump-shaped consumption tax revenue curve for consumption tax is

$$\frac{\partial c/c}{\partial \tau^c/\tau^c} < -1.$$

A necessary condition for a hump-shaped labor income tax revenue curve for labor income tax is

$$\frac{\partial n/n}{\partial \tau^n/\tau^n} < -1$$

*Proof.* In the case of the consumption tax revenue curve, it is obvious, because the consumption tax revenue is  $\tau^c c$ . It is also obvious in the case of the labor income tax revenue curve, because the labor income tax revenue is  $\tau^n wn$  and the equilibrium wage rate w is independent of  $\tau^c$  and  $\tau^n$ . (w = 1 in this static economy.)

To understand the difference in the shape of the tax revenue curves for consumption and labor income taxes, it is useful to focus on the consumption–labor supply choice optimization condition of the household:

$$-\frac{U_n}{U_c} = RPL,\tag{5}$$

where *RPL* is the relative price of leisure, which is given by

$$RPL \equiv \frac{1 - \tau^n}{1 + \tau^c} w. \tag{6}$$

An increase in consumption tax,  $\tau^c$ , or labor income tax,  $\tau^n$ , reduces the RPL, and it reduces both consumption and labor supply through the substitution effects. Proposition 1 decomposes the changes in consumption and labor supply as a product of these two effects.

**Proposition 1.** The equilibrium elasticity of consumption with respect to  $\tau^c$  is given by

$$\left|\frac{\partial c/c}{\partial \tau^c/\tau^c}\right| = \left|\frac{\partial RPL/RPL}{\partial \tau^c/\tau^c}\right| \times \left[-\frac{cU_{cc}}{U_c} + \frac{nU_{nn}}{U_n} + \frac{cU_{cn}}{U_n} - \frac{nU_{cn}}{U_c}\right]^{-1}.$$

The analog of labor supply with respect to  $\tau^n$  is given by

$$\left|\frac{\partial n/n}{\partial \tau^n/\tau^n}\right| = \left|\frac{\partial RPL/RPL}{\partial \tau^n/\tau^n}\right| \times \left[-\frac{cU_{cc}}{U_c} + \frac{nU_{nn}}{U_n} + \frac{cU_{cn}}{U_n} - \frac{nU_{cn}}{U_c}\right]^{-1}.$$

*Proof.* Taking the first-order derivative by a tax rate, the equation (5) implies<sup>3</sup>

$$\frac{\partial c/c}{\partial \tau^j/\tau^j} \left[ -\frac{cU_{cc}}{U_c} + \frac{cU_{cn}}{U_n} \right] + \frac{\partial n/n}{\partial \tau^j/\tau^j} \left[ \frac{nU_{nn}}{U_n} - \frac{nU_{cn}}{U_c} \right] = \frac{\partial RPL/RPL}{\partial \tau^j/\tau^j},$$

for j = c and n. Since y = c = n at equilibrium in this static economy, then

$$\frac{\partial y/y}{\partial \tau^j/\tau^j} = \frac{\partial c/c}{\partial \tau^j/\tau^j} = \frac{\partial n/n}{\partial \tau^j/\tau^j}.$$

Finally, the elasticities are given by

$$\frac{\partial y/y}{\partial \tau^j/\tau^j} = \frac{\partial c/c}{\partial \tau^j/\tau^j} = \frac{\partial n/n}{\partial \tau^j/\tau^j} = \frac{\partial RPL/RPL}{\partial \tau^j/\tau^j} \times \left[ -\frac{cU_{cc}}{U_c} + \frac{nU_{nn}}{U_n} + \frac{cU_{cn}}{U_n} - \frac{nU_{cn}}{U_c} \right]^{-1}$$

The first parts  $\left(\left|\frac{\partial RPL/RPL}{\partial \tau^c/\tau^c}\right|$  and  $\left|\frac{\partial RPL/RPL}{\partial \tau^n/\tau^n}\right|$ ) are the negative effects on the RPL of increasing tax rates; meanwhile, the second parts  $\left(\left[-\frac{cU_{cc}}{U_c} + \frac{nU_{nn}}{U_n} + \frac{cU_{cn}}{U_n} - \frac{nU_{cn}}{U_c}\right]^{-1}\right)$  are the sum of substitution and income effects. The substitution and income effects depend on the functional form of utility, and it is common among the cases of  $\tau^c$  and  $\tau^c$ .<sup>4</sup> The key difference comes from the elasticities of the RPL with respect to  $\tau^c$  and  $\tau^n$ , as in Proposition 2.

**Proposition 2.** The elasticities of the RPL with respect to  $\tau^c$  and  $\tau^n$  are given by

$$\left| \frac{\partial RPL/RPL}{\partial \tau^c / \tau^c} \right| = \frac{\tau^c}{1 + \tau^c},$$
$$\left| \frac{\partial RPL/RPL}{\partial \tau^n / \tau^n} \right| = \frac{\tau^n}{1 - \tau^n}.$$

$$-\frac{cU_{cc}}{U_c}+\frac{nU_{nn}}{U_n}+\frac{cU_{cn}}{U_n}-\frac{nU_{cn}}{U_c}>0.$$

<sup>&</sup>lt;sup>3</sup>The detail of the derivations is described in Appnedix A.

<sup>&</sup>lt;sup>4</sup>It is assumed that an increase in  $\tau^c$  ( $\tau^n$ ) reduces consumption (labor supply). That is,

These are increasing in  $\tau^c$  and  $\tau^n$ , respectively. Their limits are given by

$$\lim_{\tau^{c} \to \infty} \left| \frac{\partial RPL/RPL}{\partial \tau^{c}/\tau^{c}} \right| = 1,$$
$$\lim_{\tau^{n} \to 1} \left| \frac{\partial RPL/RPL}{\partial \tau^{n}/\tau^{n}} \right| = \infty.$$

*Proof.* These are obvious by  $RPL = \frac{1-\tau^n}{1+\tau^c}w$ .

Proposition 2 shows that the limit of the elasticity of the RPL to  $\tau^c$  is completely different from that to  $\tau^n$ . The elasticity of the RPL by increasing  $\tau^c$  is at most 1, whereas it becomes infinity in the case of  $\tau^n$ .

By Propositions 1 and 2, the following proposition is obtained.

**Proposition 3.** The limit of elasticity of equilibrium consumption with respect to  $\tau^c$  and that of equilibrium labor supply with respect to  $\tau^n$  are given, respectively, by

$$\begin{split} \lim_{\tau^c \to \infty} \left| \frac{\partial c/c}{\partial \tau^c / \tau^c} \right| &= \left[ -\frac{cU_{cc}}{U_c} + \frac{nU_{nn}}{U_n} + \frac{cU_{cn}}{U_n} - \frac{nU_{cn}}{U_c} \right]^{-1},\\ \lim_{\tau^n \to 1} \left| \frac{\partial n/n}{\partial \tau^n / \tau^n} \right| &= \infty, \end{split}$$

as long as  $\left[-\frac{cU_{cc}}{U_c} + \frac{nU_{nn}}{U_n} + \frac{cU_{cn}}{U_n} - \frac{nU_{cn}}{U_c}\right]$  is finite.

*Proof.* These are obvious by Propositions 1 and 2.

Proposition 3 shows that the condition for a hump-shaped Laffer curve in Lemma 1 is always satisfied in the case of  $\tau^n$ . On the other hand, it depends on the functional form of utility in the case of  $\tau^c$ .

# 4 Discussions

# 4.1 Examples of the utility function and the Laffer curve for consumption tax

In Proposition 3 in Section 3, it is shown that the condition for a hump-shaped Laffer curve in Lemma 1 is always satisfied in the case of  $\tau^n$ , whereas in the case of  $\tau^c$  it depends on the functional form of utility. This finding closely relates to that of Hiraga and Nutahara (2019), who found that the shape of the tax revenue curve for consumption tax is very sensitive to the functional form of the utility. The following examples are in line with their finding.

**Example 1: King–Plossor–Rebelo utility with constant labor supply elasticity** The necessary condition in Lemma 1 is never satisfied if the utility function is of a King–Plossor–Rebelo type with constant labor supply elasticity given by

$$U = \frac{1}{1-\eta} \Big\{ c^{1-\eta} \Big[ 1 - \kappa (1-\eta) n^{1+\lambda} \Big]^{\eta} - 1 \Big\},\,$$

as employed by Shimer (2009) and Trabandt and Uhlig (2011). In this case,

$$-\frac{cU_{cc}}{U_c} = \eta, \quad \frac{nU_{nn}}{U_n} = \lambda + \frac{(1-\eta)^2 \kappa (1+\lambda)n}{1-\kappa (1-\eta)n^{1+\lambda}},$$
$$\frac{cU_{cn}}{U_n} = 1-\eta, \quad -\frac{nU_{cn}}{U_c} = \frac{\eta \kappa (1-\eta)(1+\lambda)n^{1+\lambda}}{1-\kappa (1-\eta)n^{1+\lambda}}.$$

In our baseline model in Section 2, the closed form solution of c and n is given by

$$c = n = (1 - \tau^n)^{1/(1+\lambda)} \left[\tau^c \eta \kappa \left(1 + \lambda\right) + \kappa(\eta \lambda + 1) - \tau^n \kappa(1 - \eta)\right]^{-1/(1+\lambda)}$$

It implies that *n* is decreasing in  $\tau^c$  and  $n \to 0$  as  $\tau^c \to \infty$ . Then,  $\left[-\frac{cU_{cc}}{U_c} + \frac{nU_{m}}{U_n} + \frac{cU_{cn}}{U_c} - \frac{nU_{cn}}{U_c}\right]^{-1}$  is increasing in  $\tau^c$  and

$$\lim_{\tau^c \to \infty} \left[ -\frac{cU_{cc}}{U_c} + \frac{nU_{nn}}{U_n} + \frac{cU_{cn}}{U_n} - \frac{nU_{cn}}{U_c} \right]^{-1} = \frac{1}{1+\lambda}.$$

This implies that  $\left|\frac{\partial c/c}{\partial \tau^c/\tau^c}\right|$  cannot be greater than 1. Therefore, the Laffer curve for consumption tax is monotonically increasing.

The Laffer curves for consumption and labor income taxes are given by

$$\begin{aligned} \tau^{c}c &= \tau^{c}(1-\tau^{n})^{1/(1+\lambda)} \left[\tau^{c}\eta\kappa\left(1+\lambda\right) + \kappa(\eta\lambda+1) - \tau^{n}\kappa(1-\eta)\right]^{-1/(1+\lambda)}.\\ \tau^{n}wn &= \tau^{n}(1-\tau^{n})^{1/(1+\lambda)} \left[\tau^{c}\eta\kappa\left(1+\lambda\right) + \kappa(\eta\lambda+1) - \tau^{n}\kappa(1-\eta)\right]^{-1/(1+\lambda)}. \end{aligned}$$

**Example 2: Additively separable utility** The necessary condition for a hump-shaped Laffer curve for consumption in Lemma 1 can be satisfied if the utility function is additively separable in consumption and labor supply with constant relative risk aversion and constant labor supply elasticity given by

$$U = \frac{c^{1-\eta} - 1}{1-\eta} - \kappa \frac{n^{1+\lambda}}{1+\lambda}.$$

The relative risk aversion is  $\eta$ , and the inverse of labor supply elasticity is  $\lambda$ . In this case,

$$-\frac{cU_{cc}}{U_c}=\eta, \qquad \frac{nU_{nn}}{U_n}=\lambda, \qquad \frac{cU_{cn}}{U_n}=0, \quad -\frac{nU_{cn}}{U_c}=0,$$

and then,

$$\left[-\frac{cU_{cc}}{U_c}+\frac{nU_{nn}}{U_n}+\frac{cU_{cn}}{U_n}-\frac{nU_{cn}}{U_c}\right]^{-1}=\frac{1}{\eta+\lambda}.$$

This implies that the elasticities of consumption is increasing in  $\tau^c$  and  $\tau^n$ . Therefore, the Laffer curve for consumption tax can be hump-shaped if and only if  $\eta + \lambda < 1$ .

In this case, the closed form solution of *c* and *n* is given by

$$c = n = \left[\frac{\kappa(1+\lambda)}{1-\tau^n}(1+\tau^c)\right]^{-\frac{1}{\eta+\lambda}}$$

.

Then, the Laffer curves for consumption and labor income taxes are given by

$$\tau^{c}c = \tau^{c} \left[\frac{\kappa(1+\lambda)}{1-\tau^{n}}(1+\tau^{c})\right]^{-\frac{1}{\eta+\lambda}}.$$
  
$$\tau^{n}wn = \tau^{n} \left[\frac{\kappa(1+\lambda)}{1-\tau^{n}}(1+\tau^{c})\right]^{-\frac{1}{\eta+\lambda}}$$

#### 4.2 Extension to a dynamic model

The main results in Section 3 are obtained in a simple static general equilibrium model. The assumption of the static model is not crucial, and the main results are applicable to a dynamic version of the model with investment and capital. Appendix B shows that Propositions 1, 2, and 3 in Section 3 hold even in a dynamic general equilibrium model at a steady state.

# 4.3 Case where the tax revenue is used as the government consumption

In the baseline model, the tax revenue is used as the lump-sum transfer to the household. The main results are also applicable to the alternative fiscal policy scheme where the tax revenue is used as the government consumption;

$$g = \tau^c c + \tau^n w n.$$

In this case, the resource constraint is

$$c + g = y$$
.

Appendix C shows that the elasticities of consumption and labor supply under this fiscal policy in both static and dynamic economies. The difference in the elasticities of the RPL with respect to consumption and labor income taxes is still the key of the difference in the shapes of the Laffer curves.

#### 4.4 Implication on a heterogenous-agent model

In the model, the representative household is considered. This assumption is not crucial for the difference in the shapes of the Laffer curves for consumption and labor income taxes. Even if agents are heterogenous, each individual faces the RPL in the consumption-labor supply choice. The effects of increasing in tax rates on the RPL is common among agents. Then, the difference in the elasticities of the RPL with respect to consumption and labor income taxes is still the key.

One might think that the transfer scheme has important effects in a heterogenousagent model. Given the condition where the tax revenue is used as the government consumption as in the previous subsection, an additional transfer generates positive income effect, that increases consumption and decreases labor supply. Then, the Laffer curve for consumption tax is more unlikely to be hump-shaped, while that for labor income tax is more likely to be hump-shaped.

### 5 Concluding Remarks

Recent studies on Laffer curves (tax revenue curves) using dynamic general equilibrium models find that the Laffer curve for consumption tax might not be hump-shaped, but monotonically increasing, whereas the Laffer curve for labor income tax is humpshaped. The current study investigated the cause of this difference in shape between the two Laffer curves by decomposing the effects of an increase in tax rate on the tax base into two parts: (i) the effects on the relative price of leisure (RPL), and (ii) the substitution and income effects. It has been shown that the first effect with respect to the consumption tax rate is completely different from that with respect to the labor income tax rate, while the second effect is common among the taxes and depends on the functional form of the utility. The elasticity of the RPL by increasing the consumption tax rate is at most 1, whereas in the case of labor income tax it can be infinity as the tax rate increases.

Tax structure has received significant attention in recent years, as both political and academic issues. These issues include the effects of tax structure on tax revenue, economic efficiency, and economic performance. The main finding of this paper contributes to our knowledge in these areas.

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# Appendix

### A. Derivations in the Proof of Proposition 1

The consumption-leisure choice optimization condition is

$$-\frac{U_n}{U_c} = RPL.$$

Taking the first-order derivative by  $\tau^{j}$ , one can obtain

$$-U_{cn}\frac{\partial c}{\partial \tau^{j}} - U_{nn}\frac{\partial n}{\partial \tau^{j}} = \left[U_{cc}\frac{\partial c}{\partial \tau^{j}} + U_{cn}\frac{\partial n}{\partial \tau^{j}}\right]RPL + U_{c}\frac{\partial RPL}{\partial \tau^{j}}$$

since c and n depend on  $\tau^{j}$ . This condition is rewritten as

$$-cU_{cn}\frac{\partial c/c}{\partial \tau^{j}/\tau^{j}} - nU_{nn}\frac{\partial n/n}{\partial \tau^{j}/\tau^{j}} = \left[cU_{cc}\frac{\partial c/c}{\partial \tau^{j}/\tau^{j}} + nU_{cn}\frac{\partial n/n}{\partial \tau^{j}/\tau^{j}}\right]RPL + U_{c}RPL\frac{\partial RPL/RPL}{\partial \tau^{j}/\tau^{j}}.$$

Because the consumption-labor supply choice optimization condition is rewritten as

$$RPL = -\frac{U_n}{U_c},$$

one obtains

$$-cU_{cn}\frac{\partial c/c}{\partial \tau^{j}/\tau^{j}} - nU_{nn}\frac{\partial n/n}{\partial \tau^{j}/\tau^{j}} = -\left[cU_{cc}\frac{\partial c/c}{\partial \tau^{j}/\tau^{j}} + nU_{cn}\frac{\partial n/n}{\partial \tau^{j}/\tau^{j}}\right]\frac{U_{n}}{U_{c}} - U_{n}\frac{\partial RPL/RPL}{\partial \tau^{j}/\tau^{j}}$$

$$\iff -\frac{cU_{cn}}{U_{n}}\frac{\partial c/c}{\partial \tau^{j}/\tau^{j}} - \frac{nU_{nn}}{U_{n}}\frac{\partial n/n}{\partial \tau^{j}/\tau^{j}} = -\left[\frac{cU_{cc}}{U_{c}}\frac{\partial c/c}{\partial \tau^{j}/\tau^{j}} + \frac{nU_{cn}}{U_{c}}\frac{\partial n/n}{\partial \tau^{j}/\tau^{j}}\right] - \frac{\partial RPL/RPL}{\partial \tau^{j}/\tau^{j}}$$

$$\iff \frac{\partial c/c}{\partial \tau^{j}/\tau^{j}}\left[-\frac{cU_{cc}}{U_{c}} + \frac{cU_{cn}}{U_{n}}\right] + \frac{\partial n/n}{\partial \tau^{j}/\tau^{j}}\left[\frac{nU_{nn}}{U_{n}} - \frac{nU_{cn}}{U_{c}}\right] = \frac{\partial RPL/RPL}{\partial \tau^{j}/\tau^{j}}.$$

#### **B.** Robustness in a dynamic model

Consider a dynamic version of the model in Section 2. In this case, the capital stock  $k_t$  and investment  $i_t$  are introduced to the households' problem.

The representative households' problem is

$$\max_{\{c_t, k_{t+1}, i_t, n_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t U(c_t, n_t)$$
  
s.t.  $(1 + \tau_t^c) c_t + i_t \le (1 - \tau_t^n) w_t n_t + r_t k_t + s_t,$   
 $k_{t+1} = (1 - \delta) k_t + i_t,$ 

where  $r_t$  denotes the rental rate of capital.

The firms' problem is

$$\max_{k_t, n_t} \left[ y_t - r_t k_t - w_t n_t \right],$$
  
s.t.  $y_t = k_t^{\alpha} n_t^{1-\alpha}.$ 

The government transfers its tax revenue to the households:

$$s_t = \tau^c c_t + \tau^n w_t n_t.$$

Finally, the resource constraint of this closed economy is

$$c_t + i_t = y_t.$$

Even in this dynamic model, the key equation is still the consumption–labor supply choice condition:

$$-\frac{U_n(t)}{U_c(t)} = RPL_t,$$

where  $RPL_t = (1 - \tau_t^n) / (1 + \tau_t^c) w_t$ .

Propositions 1, 2, and 3 in Section 3 do hold in this dynamic model at a steady state, given the following Lemmas 2 and 3.

**Lemma 2.** The steady-state real wage rate, w, is independent of the consumption tax rate,  $\tau^c$ , and the labor income tax rates,  $\tau^n$ .

*Proof.* The Euler equation at a steady state is given by

$$1 = \beta \left[ 1 - \delta + \alpha \frac{y}{k} \right].$$

Then, it is obvious that the capital–output ratio k/y is independent of  $\tau^c$  and  $\tau^n$ . It implies the labor–output ratio n/y is also independent of  $\tau^c$  and  $\tau^n$  at a steady state, because the production function is rewritten as

$$1 = \left(\frac{k}{y}\right)^{\alpha} \left(\frac{n}{y}\right)^{1-\alpha}.$$

Therefore, the steady-state real wage rate w is also independent of  $\tau^c$  and  $\tau^n$ , since it is given by

$$w = (1 - \alpha)\frac{y}{n}.$$

**Lemma 3.** At a steady state, the elasticities of output, consumption, and labor supply with respect to consumption and labor income taxes have the same value:

$$\frac{\partial y/y}{\partial \tau^j/\tau^j} = \frac{\partial c/c}{\partial \tau^j/\tau^j} = \frac{\partial n/n}{\partial \tau^j/\tau^j}$$

for j = c and n.

*Proof.* As in the proof for Lemma 2, the labor–output ratio n/y is independent of  $\tau^c$  and  $\tau^n$ . Then, it is obvious that  $\frac{\partial y/y}{\partial \tau^j/\tau^j} = \frac{\partial n/n}{\partial \tau^j/\tau^j}$  for j = c and n.

Because the capital-output ratio k/y is independent of  $\tau^c$  and  $\tau^n$ , then the investmentoutput ratio i/y is also independent of  $\tau^c$  and  $\tau^n$ . Because of the resource constraint of this economy c + i = y, the consumption-output ratio c/y is also independent of  $\tau^c$  and  $\tau^n$ . Therefore,  $\frac{\partial y/y}{\partial \tau^j/\tau^j} = \frac{\partial c/c}{\partial \tau^j/\tau^j}$  for j = c and n

# C. Robustness in the case where the tax revenue is used as the government consumption

Suppose that the total tax revenue is used as the government consumption:

$$g = \tau^c c + \tau^n w n.$$

Then, the resource constraint implies

$$c + g = y$$
  
$$\iff (1 + \tau^{c})c + \tau^{n}wn = y$$
  
$$\iff (1 + \tau^{c})\left(\frac{c}{y}\right) + \tau^{n}w\left(\frac{n}{y}\right) = 1.$$

**Static model:** In the case of static model, n = y and w = 1, and then

$$(1 + \tau^{c})\left(\frac{c}{y}\right) + \tau^{n} = 1$$
$$\iff c = \frac{1 - \tau^{n}}{1 + \tau^{c}}y$$
$$\iff c = RPLy.$$

Therefore, it is obtained that

$$\frac{\partial c/c}{\partial \tau^j/\tau^j} = \frac{\partial RPL/RPL}{\partial \tau^j/\tau^j} + \frac{\partial y/y}{\partial \tau^j/\tau^j}$$

As in Appendix A, the consumption-labor supply choice condition implies

$$\frac{\partial c/c}{\partial \tau^j/\tau^j} \left[ -\frac{cU_{cc}}{U_c} + \frac{cU_{cn}}{U_n} \right] + \frac{\partial n/n}{\partial \tau^j/\tau^j} \left[ \frac{nU_{nn}}{U_n} - \frac{nU_{cn}}{U_c} \right] = \frac{\partial RPL/RPL}{\partial \tau^j/\tau^j}$$

Finally, the elasticities of consumption and labor supply are given by Proposition 4.

**Proposition 4.** Suppose that the tax revenue is used as government consumption. The equilibrium elasticity of consumption with respect to  $\tau^c$  is given by

$$\left|\frac{\partial c/c}{\partial \tau^c/\tau^c}\right| = \left|\frac{\partial RPL/RPL}{\partial \tau^c/\tau^c}\right| \times \left[1 + \frac{nU_{nn}}{U_n} - \frac{nU_{cn}}{U_c}\right] \left[-\frac{cU_{cc}}{U_c} + \frac{nU_{nn}}{U_n} + \frac{cU_{cn}}{U_n} - \frac{nU_{cn}}{U_c}\right]^{-1}.$$

The analog of labor supply with respect to  $\tau^n$  is given by

$$\left|\frac{\partial n/n}{\partial \tau^n/\tau^n}\right| = \left|\frac{\partial RPL/RPL}{\partial \tau^n/\tau^n}\right| \times \left[1 + \frac{cU_{cc}}{U_c} - \frac{cU_{cn}}{U_n}\right] \left[-\frac{cU_{cc}}{U_c} + \frac{nU_{nn}}{U_n} + \frac{cU_{cn}}{U_n} - \frac{nU_{cn}}{U_c}\right]^{-1}.$$

In this case, the substitution and income effects are no longer common. However, the difference in the elasticities of the RPL is still the key to understand the difference in the shapes of the Laffer curves. Proposition 5 shows the limits of elasticities of consumption and labor supply.

**Proposition 5.** Suppose that the tax revenue is used as government consumption. The limit of elasticity of equilibrium consumption with respect to  $\tau^c$  and that of equilibrium labor supply with respect to  $\tau^n$  are given, respectively, by

$$\lim_{\tau^c \to \infty} \left| \frac{\partial c/c}{\partial \tau^c / \tau^c} \right| = \left[ 1 + \frac{nU_{nn}}{U_n} - \frac{nU_{cn}}{U_c} \right] \left[ -\frac{cU_{cc}}{U_c} + \frac{nU_{nn}}{U_n} + \frac{cU_{cn}}{U_n} - \frac{nU_{cn}}{U_c} \right]^{-1},$$
$$\lim_{\tau^n \to 1} \left| \frac{\partial n/n}{\partial \tau^n / \tau^n} \right| = \infty,$$

as long as  $\left[-\frac{cU_{cc}}{U_c} + \frac{nU_{nn}}{U_n} + \frac{cU_{cn}}{U_n} - \frac{nU_{cn}}{U_c}\right]$  is finite.

**Dynamic model:** Focus on a steady state. In the dynamic model with the Cobb-Douglas technology, the marginal productivity condition is

$$w = (1 - \alpha)\frac{y}{n},$$

and then

$$(1 + \tau^{c})\left(\frac{c}{y}\right) + \tau^{n}w\left(\frac{n}{y}\right) = 1$$
  
$$\iff (1 + \tau^{c})\left(\frac{c}{y}\right) + (1 - \alpha)\tau^{n} = 1$$
  
$$\iff c = \frac{1 - (1 - \alpha)\tau^{n}}{1 + \tau^{c}}y.$$
  
$$\iff c = RPL^{*}y,$$

where  $RPL^* \equiv [1 - (1 - \alpha)\tau^n]/[1 + \tau^c]$ . Therefore, it is obtained

$$\frac{\partial c/c}{\partial \tau^j/\tau^j} = \frac{\partial RPL^*/RPL^*}{\partial \tau^j/\tau^j} + \frac{\partial y/y}{\partial \tau^j/\tau^j}.$$

Finally, the elasticities of consumption and labor supply are given by Proposition 6.

**Proposition 6.** Suppose that the tax revenue is used as government consumption.

The equilibrium elasticity of consumption with respect to  $\tau^c$  is given by

$$\left|\frac{\partial c/c}{\partial \tau^c/\tau^c}\right| = \left|\frac{\partial RPL/RPL}{\partial \tau^c/\tau^c}\right| \times \left[1 + \frac{nU_{nn}}{U_n} - \frac{nU_{cn}}{U_c}\right] \left[-\frac{cU_{cc}}{U_c} + \frac{nU_{nn}}{U_n} + \frac{cU_{cn}}{U_n} - \frac{nU_{cn}}{U_c}\right]^{-1}.$$

The analog of labor supply with respect to  $\tau^n$  is given by

$$\begin{aligned} \left| \frac{\partial n/n}{\partial \tau^n / \tau^n} \right| &= \left| \frac{\partial RPL/RPL}{\partial \tau^n / \tau^n} \right| \times \left[ -\frac{cU_{cc}}{U_c} + \frac{nU_{nn}}{U_n} + \frac{cU_{cn}}{U_n} - \frac{nU_{cn}}{U_c} \right]^{-1} \\ &+ \frac{(1-\alpha)\tau^n}{1-(1-\alpha)\tau^n} \left[ -\frac{cU_{cc}}{U_c} + \frac{cU_{cn}}{U_n} \right] \left[ -\frac{cU_{cc}}{U_c} + \frac{nU_{nn}}{U_n} + \frac{cU_{cn}}{U_n} - \frac{nU_{cn}}{U_c} \right]^{-1} \end{aligned}$$

The elasticity of consumption is the same as that in the static model, as shown in Proposition 4. The elasticity of labor supply is slightly different from that in the static model. However, the elasticity of the RPL with respect to the labor income tax goes to infinity as  $\tau^n \rightarrow 1$ , and then, the Laffer curve for labor income tax is naturally humpshaped. Therefore, Proposition 5 still hold even in this dynamic model.